HNSJ
Humanities & Natural Sciences Journal
ISSN: (e) 2709-0833
www.hnjournal.net
Peer-Reviewed Journal
Indexed at: NSP - SJIF
Impact Factor 2022 = 4.91

RESEARCH TITLE

GENERATING OF SQUEEZED STATE AND ENTANGLEMENT IN CAVITY OPTOMECHANICAL SYSTEM

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HNSJ, 2023, 4(5); https://doi.org/10.53796/hnsj4528

Published at 01/05/2023 **Accepted at 25/04/2023**

Abstract

We study theoretically a two-mode optomechanical system where two cavity modes are coupled to a single mechanical mode. It is shown that the whole system can yield good squeezing, which is comparable to that produced by dispersive coupling, the numerical results which we obtained showed periodic oscillations with a maximum peaks in one of the two quadrature component with a value less than 0.5. We also showed that entanglement can be achieved by calculating the variance in quadrature component of the field operators. Our results are useful in non- degenerate parametric oscillation.

Key Words: squeezed state, entanglement, quadrature component, variance, non-degenerate parametric.

عنوان البحث

توليد معامل الانضغاطية الكمية والتشابك في تجويف بصري لنظام ضوئي حركي

المستخلص

أجريت هذه الدراسة النظرية لنظام كمي مكون من حقل كهرومغناطيسي ثنائي النسق رُبط مع متذبذب ميكانيكي أحادي النسق حر الحركة. وقد تم توضيح أن النظام الكلي (المكون من الحقل والمتذبذب) يقود إلي معامل انضغاطية كمي يشابه ذاك الذي يتم الحصول عليه في حالة الاقتران المبدد للطاقة الضوئية في التجاويف البصرية. وقد أشارت النتائج التي تم الحصول عليها إلي قمم للرسم البياني لأحد مركبات المتغيرات التربيعية بقيمة قصوي أقل من 0.5. أيضاً أظهرت النتائج أنه يمكن الحصول علي التشابك الكمي وذلك بحساب التباين لمركبات المتغيرات التربيعية لمؤثرات الحقل. النتائج التي تم الحصول عليها لهذه الدراسة ذات جدوي كبيرة في التطبيقات الإلكترونية مثل معامل عدم التفسخ الحدي.

الكلمات المفتاحية: معامل الانضغاطية الكمي، التشابك الكمي، المركبة التربيعية، التباين، معامل عدم التفسخ.

I. Introduction

Squeezed states in optomechanical systems have attracted much interest and have been applied in quantum metrology and gravitational wave interferometers [**1–5**]. We are currently witnessing the emergence of a new means of production of squeezing, namely by bringing light into interaction with an optomechanical cavity, i.e. an optical cavity with one of its elements suspended so as to form on a high-quality mechanical resonator [**6–8**]. The pressure of light inside the cavity on that resonator results in optical nonlinearities described by equations that lead to the squeezing [**9-12**]. The promise of this method is the possibility to manufacture on-chip sources of squeezed light, enabling compact optical sensors and new fundamental tests of physics. In terms of applications, major results are awaited in gravitational wave detection. Although squeezed light has already been integrated into some of the detectors, it has not yet been used in actual data acquisition runs [**13–16**]. In some applications, squeezing is expected to enhance the sensitivity by up to a factor of ten. Hopefully, such a detector will not only be able to prove the existence of gravitational waves (GWs), but also provide information about their spatial distribution and temporal dynamics. This would result in a fundamentally new method for observing the universe, which has a potential to revolutionize the entire field of cavity optomechanics. In squeezed states of light, the noise of the electric field at certain phases falls below that of the vacuum state. This means that, when we turn on the squeezed light, we see less noise. This apparently paradoxical feature is a direct consequence of quantum nature of light and cannot be explained within the classical framework [**17–20**]. The basic idea of squeezing can be understood by using optomechanical systems. There are several proposals which have been done to generate and realized squeezed states of cavity fields [**1–3**]. Safavi-Naeini et al. [**4**] fabricated a micromechanical cavity resonator from a silicon microchip and observed the fluctuation spectrum at a level (4.5 ± 0.2)below the shot-noise limit despite highly excited thermal state of the mechanical resonators (104 phonons). Purdy et al. [**5**] placed a low-mass partially reflective membrane made of silicon nitride in the middle of an optical cavity and pushed the squeezing limit to 32 per cent (1.7 dB) by cooling the membrane to about 1 mK. More recently, squeezing of the cavity field is receiving considerable attention. For example, Squeezing of light using an on-resonance driving laser is achieved in [**6**]. Additional ways of producing optical squeezing in optomechanical systems have also been proposed. One example is the generation of quadrature squeezed light using the dissipative nature of the mechanical resonator in a single cavity driven by two differently detuned lasers [7]. Another is to use a double-cavity optomechanical system to generate two-mode squeezed light [**8, 9**]. In this paper, we propose an approach for the realization of squeezing for a cavity field modes coupled to a mechanical resonator by solving the master equation for the Hamiltonian. It is shown that the ultimate degree of squeezing can be obtained by numerically calculating the variance in Hermitian amplitude operators using the master equation. Furthermore, the effect of the coupling between cavity field mode coupled to a mechanical resonator and environment decoherence is effectively suppressed and significant squeezing can be achieved for the coupled system [**21**].

II. Model and Hamiltonian

The squeezed state of the electromagnetic field can be generated in many nonlinear optical processes and finds a wide range of applications in quantum information processing and quantum metrology. In this section we introduce the basic properties of single and two-mode squeezed light states. To describe squeezing in a system of a single mechanical mode coupled to a driven two cavity modes, we consider the Hamiltonian [**10**]

$$H = \sum_{k=1,2} \hbar\omega_k \, a^\dagger{}_k a_k + \hbar\omega_m b^\dagger b + \sum_{k,j=1,2} \hbar g_{k,j}(a_k + a^\dagger{}_k)\, g_k\left(a_j + a^\dagger{}_j\right)(b^\dagger + b) + \sum_{k=1,2} \hbar\varepsilon_k(a^\dagger{}_k e^{-i\omega_k t)} + a_k e^{i\omega_k t)}) \quad \_\_\_\_\_ \quad (1)$$

where (a_k) and $(a^\dagger{}_k)$ are annihilation (creation) operators of the cavity k, j, and b $(b^\dagger)$ is the phonon operators describing the oscillation of the mirror. The first two terms of Eq.(1) describe the free energy of the two optical fields as well as the mechanical oscillators with frequency ω_k and ω_m, respectively. The third term is the interaction between the cavity fields and the mechanical oscillator, where $g_{k,j}$ is the coupling strength resulted from radiation pressure, and the forth term represents the interaction between two cavity fields and the two external driving lasers with frequencies ωl1 and ωl2. Using this Hamiltonian we will examine squeezed states properties of the two optical fields a1 and a2. The state is said to be "squeezed" if it's oscillating variance become smaller than the variance of the vacuum state. In order to study the squeezing of the cavity field, we need to evaluate the variances of the generalized quadrature dimensionless operators. We introduce the Hermitian amplitude operators as

$$\hat{X}_{k=1,2} = \frac{1}{\sqrt{2}}\left(a_k + a^\dagger{}_k\right) \quad \_\_\_\_\_ \quad (2)$$

$$\hat{Y}_{k=1,2} = \frac{1}{i\sqrt{2}}\left(a_k - a^\dagger{}_k\right) \quad \_\_\_\_\_ \quad (3)$$

Where

$$\left[\hat{X}_k, \hat{Y}_{\grave{k}}\right] = i\delta k\grave{k} \quad \_\_\_\_\_ \quad (4)$$

With

$$[\hat{a}_k, a_{\grave{k}}] = \delta k\grave{k} \quad \_\_\_\_\_ \quad (5)$$

squeezed states are characterized by an uncertainty of a single emotional quadrature which is below the zero-point level. The uncertainty relation for $\hat{X}_k$ and $\hat{Y}_{\grave{k}}$ is

$$\Delta\hat{X}_k \Delta\hat{Y}_{\grave{k}} \geq \frac{1}{2} \quad \_\_\_\_\_ \quad (6)$$

The variance of the quadrature amplitude of the field operators can be obtained from

$$\langle\left(\Delta\hat{X}_k\right)^2\rangle = \langle\hat{X}_k{}^2\rangle - \langle\hat{X}_k\rangle^2 \quad \_\_\_\_\_ \quad (7)$$

and

$$\langle\left(\Delta\hat{Y}_k\right)^2\rangle = \langle\hat{Y}_{\grave{k}}{}^2\rangle - \langle\hat{Y}_{\grave{k}}\rangle^2 \quad \_\_\_\_\_ \quad (8)$$

A squeezed state of the field is obtained if the uncertainty in one of the observables satisfies the relation

$$\langle\left(\Delta\hat{X}_k\right)^2\rangle < \frac{1}{2} \quad \_\_\_\_\_ \quad (9)$$

or

$$\langle\left(\Delta\hat{Y}_k\right)^2\rangle < \frac{1}{2} \quad \_\_\_\_\_ \quad (10)$$

The state is called an ideal squeezed state if in addition to the last two equations the relation

$$\Delta\hat{X}_1\Delta\hat{Y}_1 = \frac{1}{2} \qquad (11)$$

or

$$\Delta\hat{X}_2\Delta\hat{Y}_2 = \frac{1}{2} \qquad (12)$$

is also hold [**11**].

The dynamics of the system satisfies the master equation

$$\dot{\rho} = -\frac{i}{\hbar}[\hat{H},\rho] + \sum_{j=1,2}\kappa_j\left(2a_j\rho a^{\dagger}{}_j - a^{\dagger}{}_j a_j\rho - \rho a^{\dagger}{}_j a_j\right) + \gamma(n_b+1)\,(2b\rho b^{\dagger} - b^{\dagger}b\rho - \rho b^{\dagger}b) + \gamma\, n_b(2b^{\dagger}\rho b - bb^{\dagger}\rho - \rho bb^{\dagger}) \qquad (13)$$

This equation can be numerically solved to calculate the variance for the position and phase component for the cavity fields. In the following section we will show single and two mode squeezing for the quadrature amplitudes components.

i) Single mode quadrature squeezed state

According to the Heisenberg uncertainty relation, the product of the uncertainties in determining the expectation values of two quadrature operators should be greater than or equal to their commutation value. In other words, if the variance in one of the quadrature amplitudes is smaller than the variance of the vacuum, a squeezed state of the cavity field will be obtained. In Fig. 5.1, and Fig. 5.2, we plot the fluctuation associated with the single mode and two mode component as a function of time where initially the system is in vacuum state. From Fig. 5.1a, it is obviously that the variance in one of the field quadrature amplitude, in this case the blue line, which is correspond to $\langle(\Delta\hat{X}_k)^2\rangle$, is less than 0.5. This condition shows that the field operators are squeezed. In Fig. 5.1b, we plot the quadrature amplitudes for short period of time to show their fluctuation in more resolution. The results show that if any of the two quadrature field variance is greater than 0.5, the other must be less than 0.5. Therefore, we can safely say that, the variance of one quadrature operators can decrease while the other one simultaneously increase in order to satisfy the uncertainty principle.

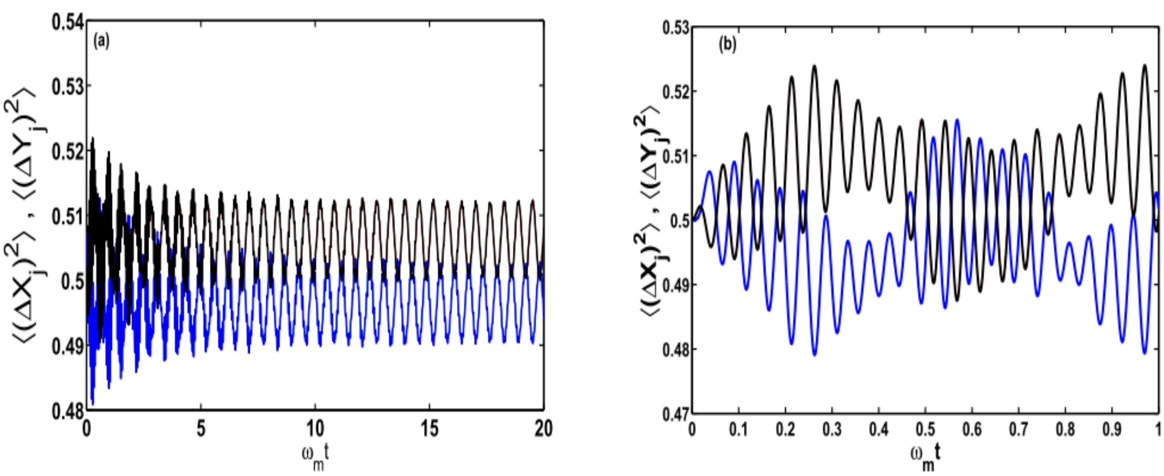

Fig: 5.1 Plot of the time evolution of variance in the cavity quadrature position and momentum operators $\hat{X}_j\ \hat{Y}_j$ where $j = 1,2$ ***The figure shows single mode squeezing*** of the cavity fields $\hat{a}_j$, the blue line is for the variance $\langle(\Delta\,\hat{X}_j)^2\rangle$, and the black line represents the the variance $\langle(\Delta\,\hat{Y}_j)^2\rangle$. The parameters are, $\omega_1 = \omega_2 = 2.5\pi\times10^8$, $\Delta1 = \Delta2 = 2.5\pi\times10^5$, Q = 6700, $\kappa_1 = \kappa_2 = 2\pi\times10^3$, $g = 4\pi\times10^6$, $\varepsilon_1 = \varepsilon_2 = 0.05g$, $\omega_m = 4\pi\times10^7$.

ii) Two-mode quadrature squeezed state

Multimode squeezed states are important since several devices produce light which is correlated at the two frequencies ω+ and ω−. Usually these frequencies are symmetrically placed either side of a carrier frequency. The emphasis of this subsection is placed on two mode squeezing found in the fields for linear combinations of the creation and annihilation operators. To show the squeezing for the two modes, we introduce the operators

$$\hat{Q} = \frac{1}{2}\left(\hat{X}_1 + \hat{X}_2\right) = \frac{1}{2\sqrt{2}}\left(a_1 + a_2 + a^{\dagger}{}_1 + a^{\dagger}{}_2\right) \quad \_\_\_\_\_\_ \quad (14),$$

and

$$\hat{P} = \frac{1}{2}\left(\hat{Y}_1 + \hat{Y}_2\right) = \frac{1}{2i\sqrt{2}}\left(a_1 + a_2 - a^{\dagger}{}_1 - a^{\dagger}{}_2\right) \quad \_\_\_\_\_\_ \quad (15),$$

with the commutation relation

$$[\,\hat{Q}, \hat{p}] = \frac{i}{2} \quad \_\_\_\_\_\_ \quad (16)$$

To determine whether the dynamics of the two mode quadrature operators produces squeezed state, we define the variance

$$\langle(\Delta\hat{O})^2\rangle = \langle\hat{O}^2\rangle - \langle\hat{O}\rangle^2 \quad \_\_\_\_\_\_ \quad (17)$$

Where ($\hat{O} = \hat{Q}\ or\ \hat{P}$). In this case the squeezing is achieved if the variance for the position or phase amplitude for the cavity fields is less than 0.25, this can be expressed according to the formulas

$$\langle(\Delta\hat{Q})^2\rangle < \frac{1}{4} \quad \_\_\_\_\_\_ \quad (18)$$

or

$$\langle(\Delta\hat{P})^2\rangle < \frac{1}{4} \quad \_\_\_\_\_\_ \quad (19)$$

where the product of their standard deviations, ΔQ and $\Delta\hat{P}$, satisfies the Heisenberg inequality

$$\Delta Q.\Delta\hat{P} \geq \frac{1}{4} \quad \_\_\_\_\_\_ \quad (20)$$

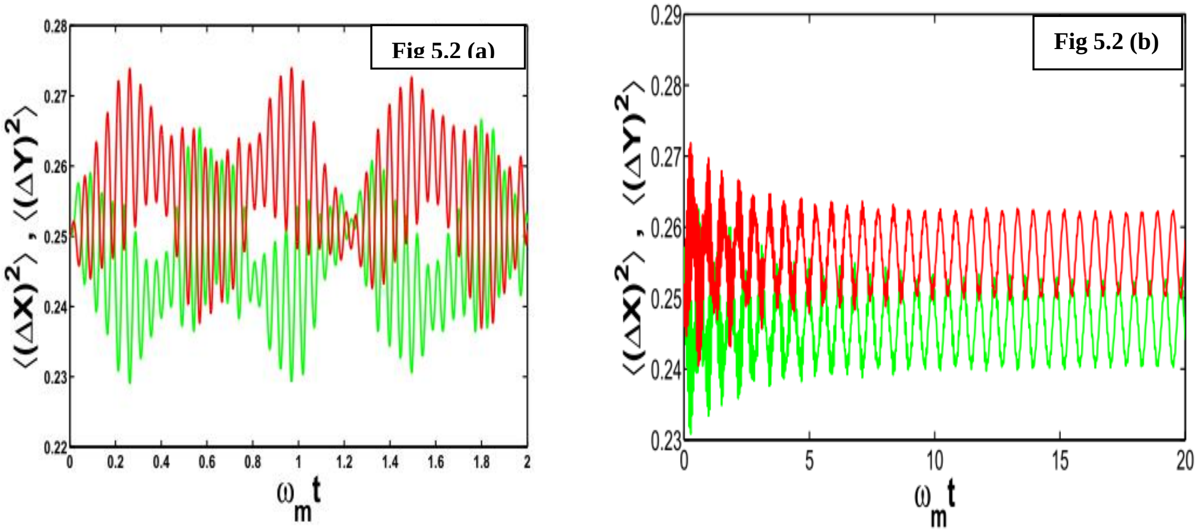

Fig 5.2, Plot of the time evolution of variance in the cavity quadrature position and momentum operators $\widehat{\mathrm{Q}}$ and $\widehat{\mathrm{P}}$. The figure shows two modes squeezing of the cavity fields $\hat{a}_k$,(k = 1,2), the green line is for the variance $\langle(\Delta\widehat{\mathrm{Q}})^2\rangle$, while the red line represents the variance $\langle(\Delta\widehat{\mathrm{P}})^2\rangle$. The parameters are the same as in Fig. 5.1.

The plot of Fig. 5.2(a) displays of the variance of the quadrature operators $\widehat{\mathrm{Q}}$ and $\widehat{\mathrm{P}}$ versus the normalized interaction time gt. The figure shows that one of the field quadrature operator's variance amplitude, in this case the green line, which is correspond to$\langle(\Delta\widehat{\mathrm{Q}})^2\rangle$, is less than 0.25 as it appear in the figure. This condition shows that the field operators are squeezed. In Fig. 5.2(b) we plot the quadrature variances for short period of time to show their fluctuations in more resolution. The results show that if any of the two quadrature field variance is greater than 0.25, the other must be less than 0.25 in order to satisfy the condition of achieving squeezed state. From Fig. 5.1 and Fig. 5.2, we can see that one of the quadrature operator's variance, for any period of the oscillations, must be less than 0.5 for single mode squeezing and smaller that 0.25 to obtain two mode squeezing. Additionally, we find that the initial state of the field has strong effect on the properties of squeezing.

III. Entanglement of the two modes of the cavity

The generation of continuous variable quantum entanglement is of great importance because it plays an essential role in quantum information theory. In this section we employ the entanglement criteria proposed by Duan [**12**]to quantify the entanglement between the two optical modes of the Hamiltonian (5.1). This criterion based on the total variance of the field quadrature operators and it provides a sufficient condition for entanglement of any two-party continuous variable states. For inseparable states in our case, the total variance of the operators ˆ u and ˆ v is required to be less than or two. The inseparability criterion can be written as

$$\langle(\Delta\hat{\mathrm{u}})^2\rangle + \langle(\Delta\hat{\mathrm{v}})^2\rangle < 2 \quad \_\_\_\_\_\_ \quad (21)$$

Where $\hat{\mathrm{u}} = (\hat{X}_1 + \hat{X}_2)$ and $\hat{\mathrm{v}} = (\hat{Y}_1 - \hat{Y}_2)$ the plot of Fig. 5.3 displays of the properties of total variance of the quadrature operators û and v̂ as a function of time. We numerically calculate the total variance of the operators û and v̂ and the time evolution shows that the total variance reduces to a value which is less than 2 for maximally entangled continuous variable state, and this agrees with the inequality (21), the value below 2 signifies the occurrence of continuous variable entanglement. Moreover, such entanglement occurs in regimes accessible to optical-fiber experiments.

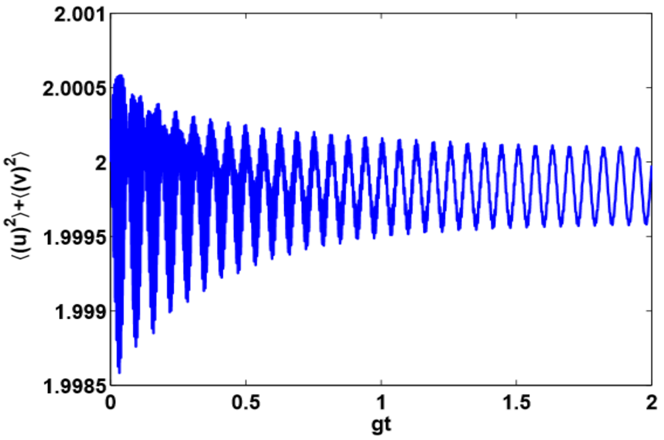

Fig 5.3 Plot of the time evolution of the total variance in the cavity quadrature position and momentum operators $\hat{u}$ and $\hat{v}$, for quantum entanglement. The parameters are the same as in Fig. 5.1 except g= $4\pi\times10^5$.

IV. Conclusion

In this paper we investigate the current states of progress in research which are squeezed and entangled states in two mode cavity with a single mode mechanical oscillator. We have shown that squeezed state can be generated in the coupled opto-mechanical system by calculating the variances in quadrature component of the field annihilation and creation operators. We obtain a numerical results for the variance versus time, the oscillations in the variance show periodic peaks with a maximum peak in one of the two quadrature component, $(\Delta Xj)^2$ in our case, is less than 0.5 for single mode, and $(\Delta Q)^2$ less than 0.25 for the two mode components. These oscillating variances show that the cavity fields are squeezed. To show the behavior of the curve in more resolution, we plot the variance for short time. The results show that the value of one of the two quadrature component is always below the fluctuation of the vacuum state, which means that the condition for achieving squeezed state is satisfied. We also note that with the time evolution of the fluctuations in the component depend on the initial state of the field. In addition, continuous variable entanglement between the two optical modes by employing Duan criteria based on the calculation of the total variance of a pair of operators is investigated. the results show that the total variance in the quadrature operators is less than 2, which is the condition of achieving entanglement. These results of entangled and squeezed states will be useful for the analysis of non-degenerate parametric oscillation [**22**].

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