HNSJ Humanities & Natural Sciences Journal
ISSN: (e) 2709-0833
www.hnjournal.net
Peer-Reviewed Journal
Indexed at: NSP - SJIF
Impact Factor 2022 = 4.91

RESEARCH TITLE

ANALYTICAL SOLUTION ON COUETTE FLOW

SULIMAN SHEEN [1] , ABDELFATAH ABASHER[2]

[1]*Deanship of the Preparatory Year,Prince Sattam bin Abdulaziz University ,Alkharj,Saudi Arabia*
EMAIL:sulimanmaleeh@gmail.com
[2]*Mathmatics Department , Faculty of Science ,Jazan Univercity,Jazan, Saudi Arabia*
EMAIL:amoaf84@gmail.com

HNSJ, 2022, 3(10); https://doi.org/10.53796/hnsj3102

Published at 01/10/2022 **Accepted at 05/09/2022**

Abstract

In this paper, we obtain basic flow solutions for stationary viscous flow between two rotating coaxial cylinders by solving the Navier -Stokes's equations in the cylindrical coordinates system (r, θ, z) for viscous incompressible fluid,simplyfied the equations and obtained analyticaly is a Zero – Order Bessel's Function in one variable.

Key Words: viscous flow, rotating, Navier -Stokes's equations , coaxial cylinders , pressure, stationary solution, perturbation equations, couette flow.

عنوان البحث

حل تحليلي لانسياب كوتي

سليمان شين[2] عبد الفتاح أبشر[1]

[1] جامعة الأمير سطام بن عبد العزيز، الخرج ، المملكة العربية السعودية

البريد الإلكتروني sulimanmaleeh@gmail.com :

[2] قسم الرياضيات ، كلية العلوم ، جامعة جازان ، جازان ، المملكة العربية السعودية

البريد الالكتروني: amoaf84@gmail.com :

HNSJ, 2022, 3(10); https://doi.org/10.53796/hnsj3102

تاريخ النشر: 2022/10/01م تاريخ القبول: 2022/09/05م

المستخلص

في هذه الورقة نحصل علي حلول الانسياب الاساسية لانسياب ثابت بين اسطوانتين متحدتين المحور بينهما مائع لا انضغاطي- لزج .ذلك بحل معادلات نايفر- استوكس في الاحداثيات الاسطوانيه. تم تبسيط المعادلات وحصلنا تحليليا على دالة بيسيل من الرتبة الصفرية في متغير واحد.

1.INTRODUCTION

The Navier -Stokes's equations for the velocity $u = (u_r, u_\theta, u_z)$ and the pressure p can be written in the form

$$\frac{\partial u_r}{\partial t} + (u.\nabla)u_r - \frac{u_\theta^2}{r} = \frac{\partial}{\partial r}\left(\frac{p}{\rho}\right) + \mathrm{v}\left(\nabla^2 u_r - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta} - \frac{u_r}{r^2}\right) \quad (1.1)$$

$$\frac{\partial u_\theta}{\partial t} + (u.\nabla)u_\theta - \frac{u_r u_\theta}{r} = \frac{\partial}{\partial \theta}\left(\frac{p}{\rho}\right) + \mathrm{v}\left(\nabla^2 u_\theta - \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r^2}\right) \quad (1.2)$$

$$\frac{\partial u_z}{\partial t} + (u.\nabla)u_z = -\frac{\partial}{\partial z}\left(\frac{p}{\rho}\right) + \mathrm{v}\nabla^2 u_z \quad (1.3)$$

in [1]

Where $(u.\nabla) = u_r \frac{\partial}{\partial r} + \frac{u_\theta}{r}\frac{\partial}{\partial \theta} + u_z \frac{\partial}{\partial z}$ (1.4)

And $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$ (1.5)

in [2]

The continuity equation in the cylindrical coordinates is given by,

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0 \quad , \quad (1.6)$$

$$V(r) = Ar + \frac{B}{r} \quad (1.7)$$

In [3]

These aforementioned equations allow a stationary solution of the form

$$u_r = u_z = 0 \;\; and \quad u_\theta = r\Omega(\mathrm{r}) \quad (1.8)$$

Thus, Navier -Stokes's equations

Reduced to $\frac{d}{dr}\left(\frac{p}{\rho}\right) = \frac{V^2}{r}$ (1.9)

and $\mathrm{v}\left(\nabla^2 V - \frac{V}{r^2}\right) = \mathrm{v}\frac{d}{dr}\left(\frac{d}{dr} + \frac{1}{r}\right)V = 0$ (1.10)

in [3]

2.THE PERTURBATION EQUATIONS AND THE NORMAL MODE

In order to investigate the solutions of the flow system described by equations (1.9) We consider an infinitesimal of the basic flow is given by (1.8) by assuming that the perturbed flow is given by

$$u_r, \quad V + u_\theta \,, u_z \quad and \qquad \varpi = \frac{\delta p}{\rho} \tag{2.1}$$

Assuming that the various perturbations are axisymmetric and independent of θ, and $v = 0$ (ideal fluid − water).From (1.1) - (1.3) we gain the following linearized equations as

$$\frac{\partial u_r}{\partial t} - 2\frac{V}{r}u_\theta = - \frac{\partial \varpi}{\partial r} \tag{2.2}$$

$$\frac{\partial u_\theta}{\partial t} + \left(\frac{dV}{dr} + \frac{V}{r}\right)u_r = 0 \tag{2.3}$$

In [3]

and
$$\frac{\partial u_z}{\partial t} = -\frac{\partial \varpi}{\partial z} \tag{2.4}$$

where ∇^2 is defined by

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \tag{2.5}$$

And the equation of continuity reduces to

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial^2 u_z}{\partial z^2} = 0 \tag{2.6}$$

By analyzing the disturbance into normal modes. We assume that the disturbances are of the following form

$$\left.\begin{array}{ll} u_r = e^{pt}u(r)coskz; & u_z = e^{pt}\omega(r)sinkz \\ u_\theta = e^{pt}v(r)coskz & \varpi = e^{pt}\varpi(r)coskz \end{array}\right\} \tag{2.7}$$

in[4]

where k is the wave number of the disturbance in the axial direction, and *p* is a constant which can be complex.

Substituting (2.7) in equations (2.2) - (2.6), we get

$$-pu + 2\frac{V}{r}v = \frac{d\varpi}{dr} \tag{2.8}$$

$$-pv - (D_* V)u = 0 \tag{2.9}$$

$$p\omega = k\varpi \tag{2.10}$$

$$\nabla^2 = \left(\frac{d}{dr} + \frac{1}{r}\right)\frac{d}{dr} - k^2 = D_* D - k^2 = DD_* + \frac{1}{r^2} - k^2, \tag{2.11}$$

$$D_* u = -k\omega \qquad (2.12)$$

In[3]

$$then \qquad \omega = -\frac{D_* u}{k} \qquad (2.13)$$

substitute ω in equation (2.10), we obtain

$$D_* u = -\frac{k^2}{p}\varpi \qquad (2.14)$$

From equation (2.14), we find

$$\varpi = -\frac{p}{k^2} D_* u \qquad (2.15)$$

Substituting (2.15) in equation (2.8), yields

$$\frac{p}{k^2} D(D_* u) - pu + 2\frac{V}{r}v = 0 \qquad (2.16)$$

By multiplying equation (2.9) by $(2\frac{V}{r})$, and multiplying equation (2.16) by P

We obtain,

$$-2p\frac{V}{r}v - 2\frac{V}{r}(D_* V)u = 0 \qquad (2.17)$$

$$\frac{p^2}{k^2}(DD_*)u - p^2 u + 2p\frac{V}{r}v = 0 \qquad (2.18)$$

Now, summation equation (2.17) to equation (2.18), we obtain

$$\frac{p^2}{k^2}(DD_*)u - p^2 u - 2\frac{V}{r}(D_* V)u = 0 \qquad (2.19)$$

But, $D_* = \frac{d}{dr} + \frac{1}{r}$, therefore $(DD_*) = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}$ - $\frac{1}{r^2}$ (2.20)

$$\frac{p^2}{k^2}\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2}\right)u - p^2 u - 2\left(A + \frac{B}{r^2}\right)\left[\left(\frac{d}{dr} + \frac{1}{r}\right)\left(Ar + \frac{B}{r}\right)\right]u = 0 \qquad (2.21)$$

angular velocity is

$$\Omega(\mathrm{r}) = A + \frac{B}{r^2}, \quad \Phi(r) = 4A\left(A + \frac{B}{r^2}\right) \qquad (2.22)$$

in[1]

where Φ(r) is a real function

By Substituting (2.22) at equation (2.21) We get:

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} - k^2u + \frac{k^2}{p^2}\Phi u = 0 \qquad (2.23)$$

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} + \left\{\left\{k^2\left(\frac{\Phi}{p^2} - 1\right) - \frac{1}{r^2}\right\}u = 0 \qquad (2.24)$$

Equation (2.24) is a Zero – Order Bessel's Function, with the solution

$$u = b_1 J_0(\gamma r) + b_2 Y_0(\gamma r) \qquad (2.25)$$

in[5]

$$\text{where,} \quad \gamma = k\sqrt{\frac{\Phi}{p^2} - 1} \qquad (2.26)$$

3.RESULT AND CONCLUSION

(1) Bessel's Function are closely associated with problems processing circular or cylindrical symmetry, because of their close association with cylindrical domains.in[5]

(2) The solutions of Bessel's equation are called cylinder functions. Bessel's Function of the first kind and second kind are special cases of cylinder functions.

(3)b_1 and b_2 are constants of integration in Equation (2.25), and J_0 and Y_0

are the Bessel functions of the first and the second kind.

The term Y_0 in the solution (2.25) is absent because it is divergent , that means

either $Y_0 \rightarrow 0$ or the constant $b_2 \rightarrow 0$, in this case we have chose $b_2 \rightarrow 0$

We then have the solution

$$u = b_1 J_0(\gamma r) \qquad (2.27)$$

REFERENCES

[1] S. Chandrasekhar. '**Hydrodynamic and Hydromagnetic Stability**', Dover, New York, 1961.

[2] Murray R.Spiegel , "**Vector Analysis and An Introduction To Tensor Analysis**",Rensselaer Polytechnic Institute ,1959.

[3] Hua-Shu Dou, Boo Cheong Khoo2 , and Khoon Seng Yeo," **Instability of Taylor-Couette Flow between Concentric Rotating Cylinders"**, Inter. J. of Thermal Science, 2008.

[4] Kyungyoon Min and Richard M. Lueptow," **Hydrodyna ic stability of viscous flow between**

rotating Porous cylinders with radial flow", Department of Mechanical Engineering, Northwestern University, Evanston, Illinois 60208 (Received 28 January 1993; accepted 1 September 1993.

[5]LARRY C. ANDREWS , **'Special functions of mathematics for engineers'**, Bellingham, Washington USA ,1998.